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Synchrotron Radiation Issues for 87.5 TeV proton beam.

One of possible implementations of a future post-LHC proton collider uses so called "high field" approach with magnetic field about or higher than 10 T. At this magnetic field level and energy higher than about 4 TeV, synchrotron radiation (SR) of protons modifies beam particle motion and can impose certain limitations on the accelerator implementation. That's why it is useful to make a preliminary analysis of SR properties taking into the account expected parameters of the collider. The starting point for the analysis has been established by the discussion at the first meeting on VLHC study at Fermilab held in November 2000 at FNAL. Presented accelerator parameter list (see the table below) claims proton energy of **87.5 TeV** in each of colliding beams with the expected luminosity of $\mathbf{10^{35}\ cm^{-2}sec^{-1}}$ per interaction region.

Table 1

Parameter	**Value**
Ring circumference C	241 km
Injection energy E_i	20.0 TeV
RMS normalized emittance at injection e_{Ni}	2.0 π·mm·mrad
Final energy E	87.5 TeV
RMS normalized emittance e_N	0.5 π·mm·mrad
Dipole magnetic field B	10 Tesla
Relativistic factor g	93284
Number of bunches k	81106
Bunch spacing t	9 nsec
Number of protons per bunch N	$1.38\cdot10^{10}$
Beta-function at the interaction point b^*	25 cm

I. Main Proton Beam Properties

This part is to justify the parameters in the table 1 and to understand main beam properties not shown there.

To find number of particles that can give required **luminosity**, let's make a simple estimate. According to [1], in the simplest case of the round beam, luminosity L for one crossing can be found from the expression:

$$L = f \cdot \frac{N^2}{4 \cdot p \cdot s^2} \qquad /1/$$

where N is number of protons per bunch, f is the frequency of bunch collision, and s is a characteristic beam radius for Reyleigh particle distribution in the beam cross-section

$$dn = \frac{N}{s^2} \cdot e^{-r^2/2\cdot s^2} \cdot r \cdot dr \qquad /2/$$

About 39% of the total number of particles in the beam are inside the cylinder with $\boldsymbol{r} = s$. This parameter s can be expressed through the beam emittance e that corresponds to the same fracture of beam current and beta-function b^* in the interaction region ([1], p. 83):

$$s^2 = \frac{e_N \cdot b^*}{g} \qquad /3/$$

Then (1) can be rewritten as

$$L = \frac{N^2 \cdot g}{4p \cdot t \cdot e_N \cdot b^*}, \qquad /4/$$

where $t = \mathbf{1/f}$ is bunch spacing. It worth to mention here that normalized emittance e_N in this expression refers to **area in phase space divided by** p.

Using data from the table 1, it is easy to calculate **number of particles per bunch** that corresponds to the planned luminosity:

$$\boldsymbol{N = 1.23 \times 10^{10}}.$$

Although we did not take into the account beam crossing angle, it is well compared to the number in the table above (1.38 10^{10}). Circulating current is about 0.2 A. The **total energy** accumulated in each of proton beams of a collider can be easily calculated as we know number of bunches ***k*** in each ring

$$W = kN \times m_0\, g \times c^2$$

That gives **W = 14 10^9 Jowls** of energy in each beam. Particle loss due to beam scattering leads to a heat load that can be roughly estimated if to take into the account beam life, which is about 10 hours. If no protection measures are undertaken, the beam energy loss rate of approximately **400 kW** or **1.6 W/m** is expected. As a result of scattering, a secondary nuclear reaction process in magnet body can provide an additional energy deposition into magnet. This issue is out of the scope of this note and requires separate investigation.

Geometrical properties of a proton beam (beam size and particle angular spread) are defined by a β-function and beam emittance. For our estimate purpose only, we will need to know maximal angular spread of protons in the beam:

$$z'_{\max} = \sqrt{\frac{6 \cdot e_N}{g \cdot b_{\min}}} \qquad /5/$$

where coefficient 6 is introduced to take into the account 95% of the beam particles. Taking emittance and relativistic factor at injection, we can calculate z'_{max} = 2 10^{-6}.
Beam size can be estimated using the known expression similar to /5/:

$$z_{\max} = \sqrt{\frac{6 \cdot e_N \cdot b_{\max}}{g}} \qquad /6/$$

that gives $z_{max} \approx 0.5$ mm at injection, ~0.2 mm at final energy, and ~0.1 mm after betatron oscillation damping (see below).

Having the total number of particles $kN \times$, it is possible to understand the level of synchrotron radiation to take into the account.

II. Syncrotron Radiation Parameters

Synchrotron radiation power for one charged particle is [1]:

$$P = \frac{2}{3} \cdot \frac{e^2 \cdot c \cdot g^4}{4pe_0 \cdot r^2}, \qquad /7/$$

where ***e* = 1.6*10^{-19} K** is electrical charge of a particle, ***c* = 3*10^{8} m/c** is speed of light, ***e$_0$* = 8.85*10^{-12} F/m** - permittivity of free space, g » ***E/(m$_0$·c^2)*** is relativistic factor, and $r \approx \frac{m_0 c g}{eB}$ is radius of curvature of particle with energy ***E = m$_0$gc^2*** in magnetic field ***B***.
With E = 87.5 TeV and B = 10 T, for protons (m_0 = 1.673*10^{-27} kg), ρ ≈ 29197 m. Using /7/, energy loss per one turn can be calculated as

$$U_0 = P\frac{2pr}{c}, \qquad /8/$$

that gives ***U$_0$*** ≈ 15.6 MeV/turn. This energy loss must be compensated by an accelerating station. Total radiated power per one proton ring

$$P_{tot} = P \cdot kN = \frac{kN}{6 \cdot p \cdot e_0} \cdot \frac{e^4}{m_0^2 \cdot c} \cdot g^2 \cdot B^2 \qquad /9/$$

It is about 4.1 MW per one proton ring. Totally 8.2 MW of power is necessary to remove in the form of heat; the same amount of power must be compensated in the form of accelerating field. For 1 m of orbit, synchrotron radiation power (per one ring) can be calculated as:

$$P_1 = kN \cdot \frac{m_0}{12p^2} \cdot \frac{g \cdot e^5 B^3}{m_0^3}, \qquad /10/$$

that gives **P_1 = 22.2 W/m** per one ring. It is useful to notice that this number scales linearly with number of particles while luminosity scales quadratically! So, if we reduce number of particles to lower the radiated power, luminosity will decrease with much higher rate. **The way to reduce the total number of particles without compromising luminosity can be to increase bunch spacing (use less bunches) and compensate for this increase putting more protons in each bunch.**

The radiation spectrum of SR is well defined. For our purpose, we need to know wavelength that corresponds to maximum value of a spectral power density:

$$l_m = 0.42 \cdot l_c = 0.56p \cdot R \cdot g^{-3} \qquad /11/$$

After some transformation, it is possible to write down:

$$l_m = 5.5 \cdot B^{-1} \cdot g^{-2} \qquad /12/$$

that gives l_m » **0.06 nm = 0.6 Å**. Corresponding photon energy can be calculated using the expression:

$$w_m = h \cdot n_m \qquad /13/$$

where $n_m = c/l_m$ and h = 6.625·10^{-34} J·sec is the Plank constant. This energy is about 20 kV, that belongs to the soft Roentgen part of electromagnetic spectrum.

Angular distribution for synchrotron radiation is well known although it is not too easy to describe it as an explicit function. It depends on wavelength and particle energy, but it is possible to tell approximately for the whole radiated spectrum that angle of a cone that contains most of radiated power is about a » g^{-1} » $\mathbf{10^{-5}}$. Because, as it was found earlier, betatron oscillation induced synchrotron radiation spread is small if compared with a, equilibrium orbit will be used to define SR propagation patterns.

Betatron oscillation radiation damping is one of the most significant SR effects. Because of synchrotron radiation, beam betatron oscillation will decay with time, and beam emittance will become smaller. The time scale of the radiation damping is given by the expression

$$t_0 = E/P = \frac{2pr}{c} \cdot E/U_0, \qquad /14/$$

and appears to be about 1 hour. Vertical and horizontal betatron oscillation damping time constant $t \approx 2t_0$ is about 2 hours, which is quite comparable with beam life time. After damping, beam radial emittance can be estimated using the expression [1]:

$$e_{Nx} \approx 0.66 \cdot \left\langle \frac{D^2}{b} \right\rangle \cdot \frac{w_c}{mc^2} \qquad /15/$$

where D is lattice dispersion and w_c is photon energy corresponding to SR critical wavelength that can be found using /11 - 13/ and is about 8 keV. Making an evaluation of the D^2/β using data available from [2], we calculate e_{Nx} » **0.4 mm**x**mrad,** which is close to the number in the table above.

III. Synchrotron Radiation Flux Management.

If not removed out of a beam pipe, synchrotron radiation can result in vacuum degradation or even in quenching of a superconducting magnet. For proton machines, this issue was addressed first during the SSC design study [3], and then developed for the LHC [4]. The most serious problem connected to SR is vacuum degradation that leads to beam neutralization, pressure avalanche, and multifactoring. To resolve these problems, a perforated beam screen maintained at a temperature below 20 K and intercepting the radiation was introduced in LHC. Last study made for the (50+50) TeV high field VLHC [5] has shown that beam screen concept is still working in this case, although power per one meter of magnet length is close to its ultimate limit (1.6 W/m). So, a new method of SR power removal can be a path to higher accelerator energies. There is always an obvious way to solve the problem by using magnets with gap in windings that allow the radiation out of the magnet like it is being done for electron machines. Nevertheless, even for the traditional magnet design, there is a way to solve the problem.

As it was mentioned earlier, SR is tangent to particle trajectory and its intrinsic divergence is very small. For a 20-meter magnet, it is only about 0.4 mm. This makes it possible to talk about channeling of the SR through magnet beam pipe.

Let's imagine dipole magnets with length L_m distributed along the accelerator orbit with a space of the length L_s between them. Because of a very high energy, particle orbit sag inside each magnet is rather small, and it is natural to use straight magnets located so that the distance from the orbit to the magnet center line is minimal. The maximal distance from the orbit to the center line in this case is

$$s = \frac{L_m^2}{16\,r}\,, \qquad /16/$$

which is about 0.2 mm for a 10-m magnet. Each magnet rotates charged particle velocity by the angle $y_m = \frac{L_m}{r}$. For L_m = 10m, y_m » $3.4 \cdot 10^{-4}$, which is much higher than the SR spread. Then we will just neglect this spread and try to figure out the restrictions that must be imposed onto magnet design so that the SR could be removed outside magnet beam pipe. In the simplest case when there are no space between the magnets, distance ***h*** from the SR ray emitted at the beginning of a magnet to the magnet axis can be found using

$$h = \frac{L_m^2}{2\,r} \qquad /17/$$

If this distance exceeds magnet aperture, SR hits a beam pipe. If to take ***h*** = ***A*** = 20 mm, magnet length that meets the requirement that all the SR generated inside the magnet is channeled out of it is about 34 m. But this SR will hit walls of the next magnet. To avoid it, we need to use shorter magnets with intercepts between them. Some details of the intercept design will be discussed later. At this point we just need to know that these intercepts must be placed in the beam pipe at the distance ***d*** from the orbit and be cooled with an appropriate coolant. Distance ***d*** can be found using a simple geometrical constrain that SR radiated near the end of the first magnet and coming through an intercept diaphragm in front of the third magnet must reach the end of the third magnet without hitting its wall. Only orbit curvature must be taken into the account in this case. Scheme shown in Fig.1 can be used to make needed estimate.

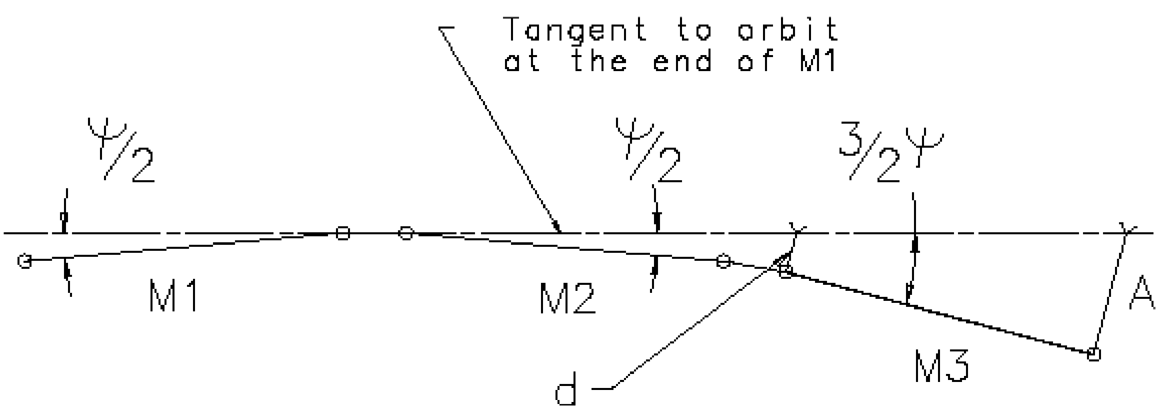

Fig. 1. SR propagation relative to the bending magnet axis

Source of light emission is at the end of the magnet M1. The light goes through the dipole M2, diaphragm ***d,*** and magnet M3 towards the output aperture ***A***. The diaphragm ***d*** must intercept all the radiation that would not fit the diaphragm ***A***. This condition allows us to write down an equations that defines maximal magnet length and needed diaphragm size:

$$A = 2\frac{L_m^2}{r} + \frac{L_m L_s}{r} \qquad /18/$$

$$d = \frac{L_m^2}{2r} + \frac{L_m L_s}{r} \qquad /19/$$

Equation /18/ allows us to calculate maximal length of magnet if we know magnet aperture ***A*** and the space between magnets $\boldsymbol{L_s}$.

$$L_m \approx \sqrt{\frac{Ar}{2}} - \frac{L_s}{4}$$

With $A \approx 20$ mm and $L_s \approx 1$ m , we have $L_m \approx 16.8$ m. Diaphragm size d can be found using /19/:

$$d \approx \frac{A}{4} + \frac{3}{4} Ls \sqrt{\frac{A}{2r}}$$

that gives $\boldsymbol{d} \approx 5.4$ mm. This is minimal distance from diaphragm to the particle orbit with magnet length of 16.8 m.

Each intercept device must handle about 370 W of power in the form of electromagnetic radiation with characteristic wavelength of about **0.5 Å.** This radiation can be readily absorbed by a rather thing layer of material. For iron, for example, characteristic absorption length for this wavelength is about 0.04 g/cm^2, or 0.05 mm of material thickness. To remove this power from the material, it is possible to use a liquid coolant like water or nitrogen, which matches in a better way better with cryogenic environment. If to use 70 K nitrogen and allow 5 K coolant temperature rise, we need 2.5 liter/min of the coolant flow. At the first glance, that does not impose a significant problem (except the total number of magnets is very high). Nevertheless, taking into the consideration a very narrow radiated power spatial distribution (about 0.2 mm thick), we find that we can not just place a cooled plate to intercept the beam. If we use a copper plate with thickness of 0.2 mm cooled from the other side, temperature rise through the plate thickness would be about 160° C. It is necessary to significantly develop surface exposed to the radiation. One of many possible approaches is shown in the picture below.

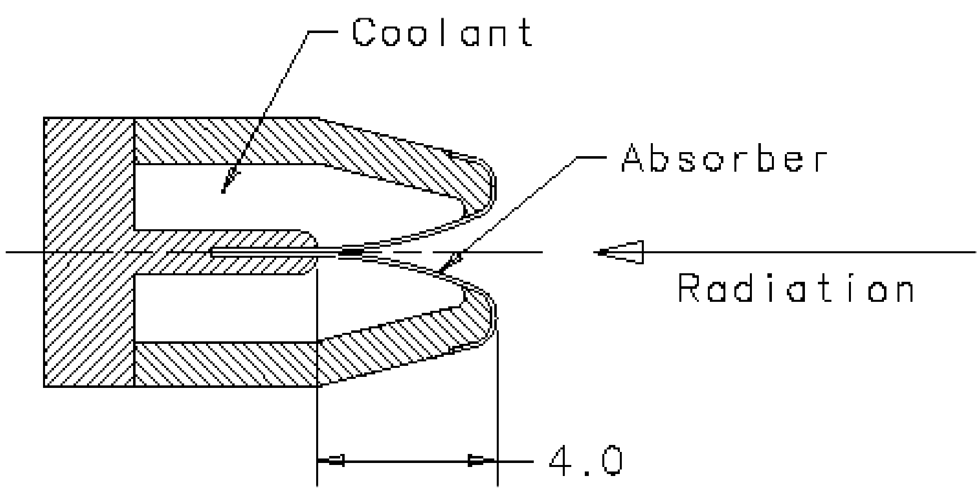

Fig. 2. Schematic of an intercept design

Thin absorber foil is placed in a way that increases an area of exposure to the SR radiation. Because of a small beam size, only the innermost part of the absorber foil is exposed to radiation. In this area, angle between radiation direction and absorber foil is rather small. Besides the whole device can be placed at a small angle to the orbit. Both measures, if implemented properly, can significantly increase exposed surface preventing excessive absorber temperature rise. For example, if curvature radius of the absorber foil is about 8 mm, only 2.5 mm of the absorber perimeter is exposed. If the active length of the device is 150 mm, exposed surface is 375 mm^2, and temperature rise for a 0.1 mm thick stainless steel foil is about 10 K, which looks quite acceptable, and even leaves some space for future optimization.

This future optimization work must justify a foil material choice, foil thickness, coolant flow details. The device's size must be small enough to prevent it from being in a close proximity to the beam. Nevertheless, even an active zone size is about 4.0 mm as in the picture above, particle beam can be rather close to the device. This can be a problem from the point of view of beam pipe impedance, so it must be also taken into consideration. For example, a proper material choice can be done that have an acceptable thermal conductivity and low electrical conductivity to reduce Q-value for undesirable harmonics. Nevertheless, some conductivity is still needed for the purpose of discharging.

Conclusion:

1. With a proper choice of magnet length, it is possible to avoid heating of magnet beam line with synchrotron radiation.
2. It is possible to suggest an intercept design that is able to handle power in the desired range. Nevertheless, problem study and modeling are needed to make a reliable design.
3. Beam pipe impedance will change at the place when intercept is located. This problem also needs a thorough study in order to optimize an intercept design.

<u>References:</u>

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[5] P. Bauer, C. Darve, T. Peterson, B. Jenninger Very Large Hadron Collider Beam Screen Design. Preliminary Investigation of Space Requirements. TD-99-032